GCSE Mathematics

Delta Paper 3:

Paper 3H (Calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

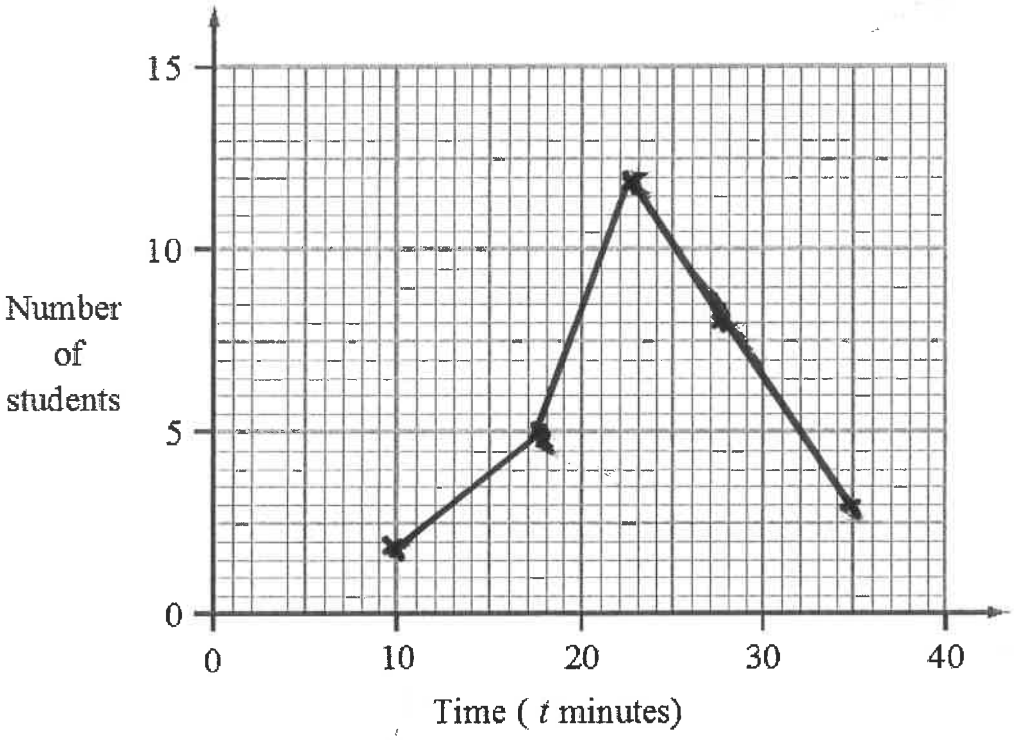
- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1. 30 students ran in a cross-country race.

The table below shows information about the times recorded for each student.

Time (t minutes)	Frequency	fx
10 $5 < t \leq 15$	2	20
17.5 $15 < t \leq 20$	5	87.5
22.5 $20 < t \leq 25$	12	270
27.5 $25 < t \leq 30$	8	220
35 $30 < t \leq 40$	3	105
	30	702.5

a) On the grid, draw a frequency polygon to show this information [2]



b) Calculate an estimate for the mean time taken for a student to complete the race.

$702.5 \div 30 = 23.41\dot{6}$ [3]

(Total 5 marks)

2. A patio tile is made up of a square **and** a semi-circle.

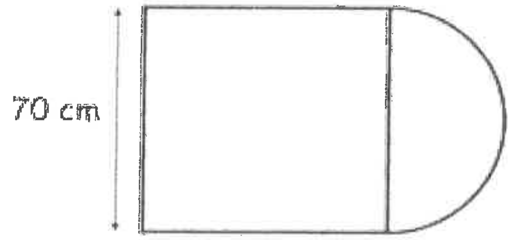
The length of the side of the square is 70cm.

Calculate the area of the whole patio tile.

$$70^2 + \frac{\pi \times 35^2}{2} =$$

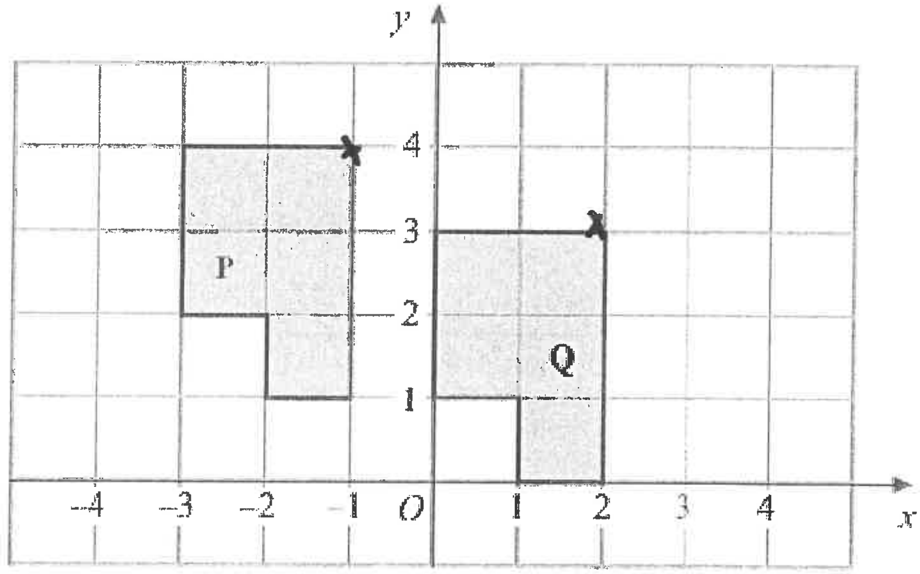
$$4900 + 1924.2255$$

$$= 6824.2255\text{cm}^2$$



(Total 2 marks)

3.



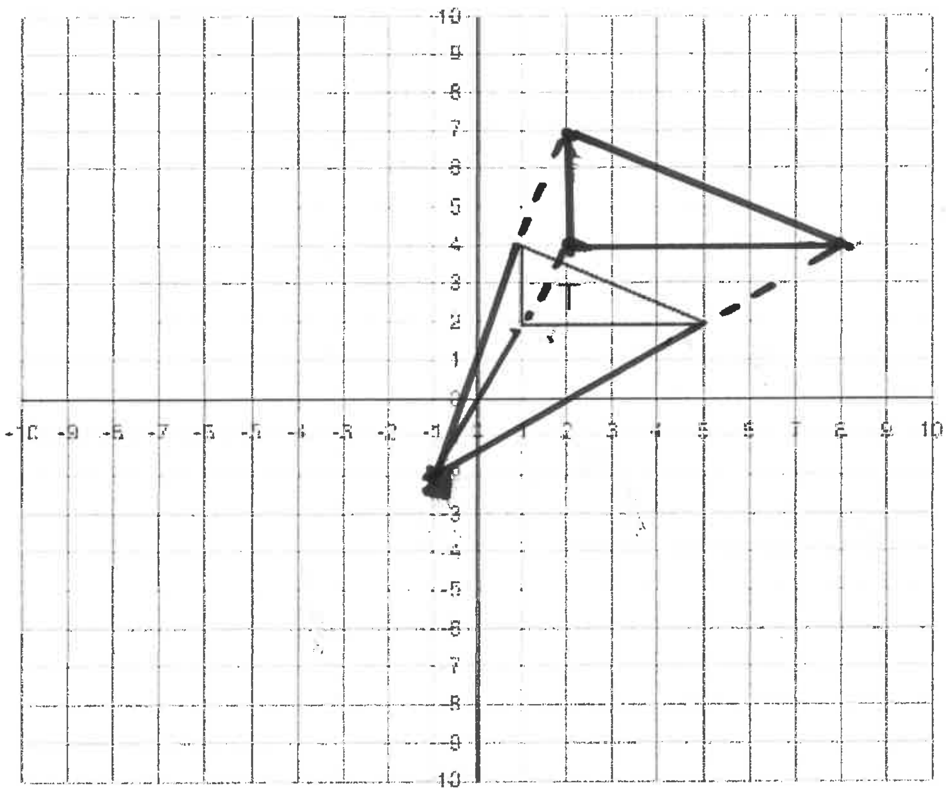
Describe fully the single transformation that will map shape **P** onto shape **Q**

translation of vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

(Total 2 marks)

4. The vertices of triangle T are (1, 2), (5, 2) and (1, 4).

Enlarge triangle *T* by scale factor $\frac{3}{2}$, with (-1, -2) as the centre of enlargement.

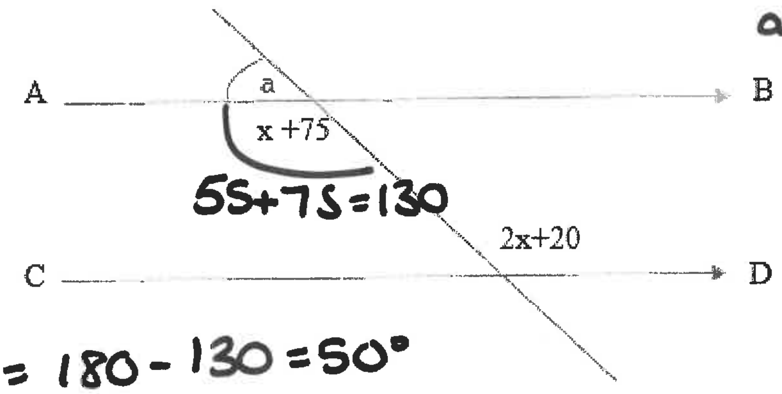


(Total 3 marks)

---

5. AB is parallel to CD.
Calculate the size of angle a.
Give reasons for your answer.

$x+75 = 2x+20$
as ~~corresponding~~ alternate angles are equal
$75 = x+20$
$x = 55$

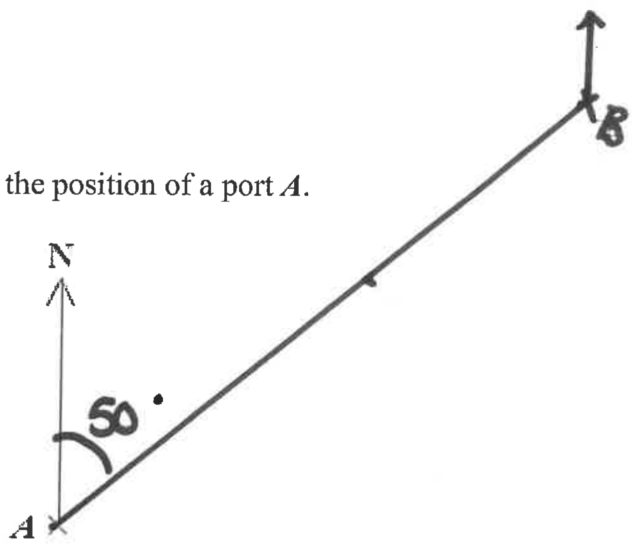


$55+75=130$

$a = 180 - 130 = 50°$
angles on a straight line add up to 180°

(Total 3 marks)

6. Here is part of a map showing the position of a port A.



B is a lighthouse 36 km from A on a bearing of 050°

(a) (i) On the diagram above using a scale of 1 cm = 4 km, show the position of B.

(ii) Write down the bearing of A from B.

$180 + 50 = 230°$

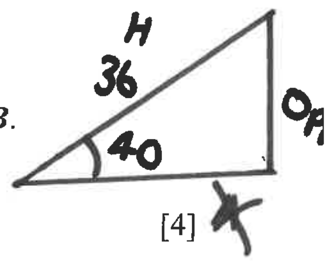
[3]

From the lighthouse at B, ships can be seen when they are within a range of 23 km of B.

A ship sails due East from A.

(b) Show, **by calculation**, that this ship will not be seen from the lighthouse at B.

You must not use a scale drawing.



$\sin 40 = \frac{Opp}{36}$ ~~≈28~~ $Opp = 23.14$

[4]

The ship will not be seen from the lighthouse because the shortest distance it would be 23.14 km.

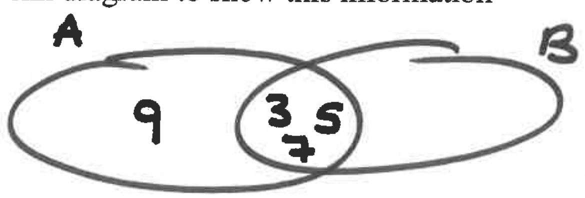
(Total 7 marks)

---

7. If $\varepsilon = \{3, 4, 5, 6, 7, 8, 9\}$ and
A = { *odd numbers* }
B = { *prime numbers* }

(a) Draw a Venn diagram to show this information

[2]



(b) Find the probability of P(A ∪ B).

[2]

$\frac{4}{7}$

(Total 4 marks)

---

8. Show that the equation below can be rearranged as $x = \frac{z(y+1)}{y-2}$

$$z + 2x = y(x - z)$$

$x(y-2) = z(y+1)$ $\quad xy - yz = z+2x$

$xy - 2x = yz + z$ $\quad y(x-z) = z+2x.$

(Total 2 marks)

---

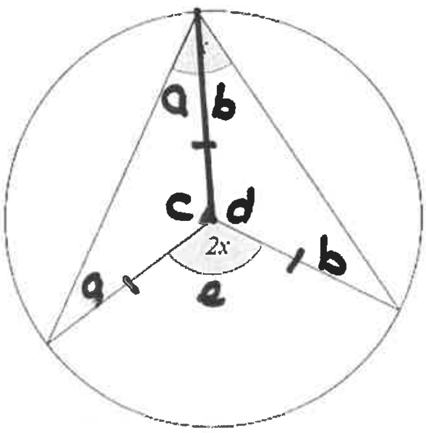
9. A class consists of 13 boys and 14 girls. One boy and one girl are chosen at random. How many combinations are there of selecting these two students.

182

(Total 1 mark)

---

10. Trevor has learned that "the angle at the centre is twice the angle at the circumference" as shown in this diagram.



Prove that the above rule is true.

$2a + c = 180°$
$+$
$2b + d = 180$

$2a + 2b + c + d = 360°$

$c + d + e = 360°$

$2a + 2b + c + d = c + d + e$

$2a + 2b = e$

$2(a+b) = e$

$a + b = x \quad e = 2x$

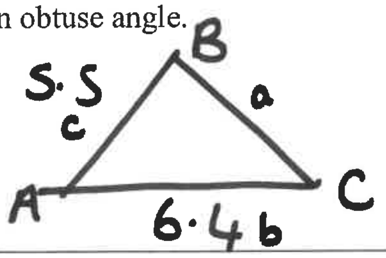
(Total 3 marks)

---

11. A triangle ABC is such that AB = 5.5cm, AC = 6.4cm and its area is 13.2cm^2.

You are told that the angle between the two known sides, angle BAC is **obtuse**.

Find this unknown obtuse angle.



Area $= \frac{1}{2} bc \operatorname{Sin} A$

$13.2 = \frac{1}{2} \times 5.5 \times 6.4 \times \operatorname{Sin} A$

$\frac{13.2}{17.6} = \operatorname{Sin} A$

(Total 3 marks)

$\operatorname{Sin} A = 0.75$

$A = 48.59$

Obtuse $180 - 48.59 = 131.41°$

12 (a) Complete the table below, for this equation which links velocity, v in mph to time t in minutes.

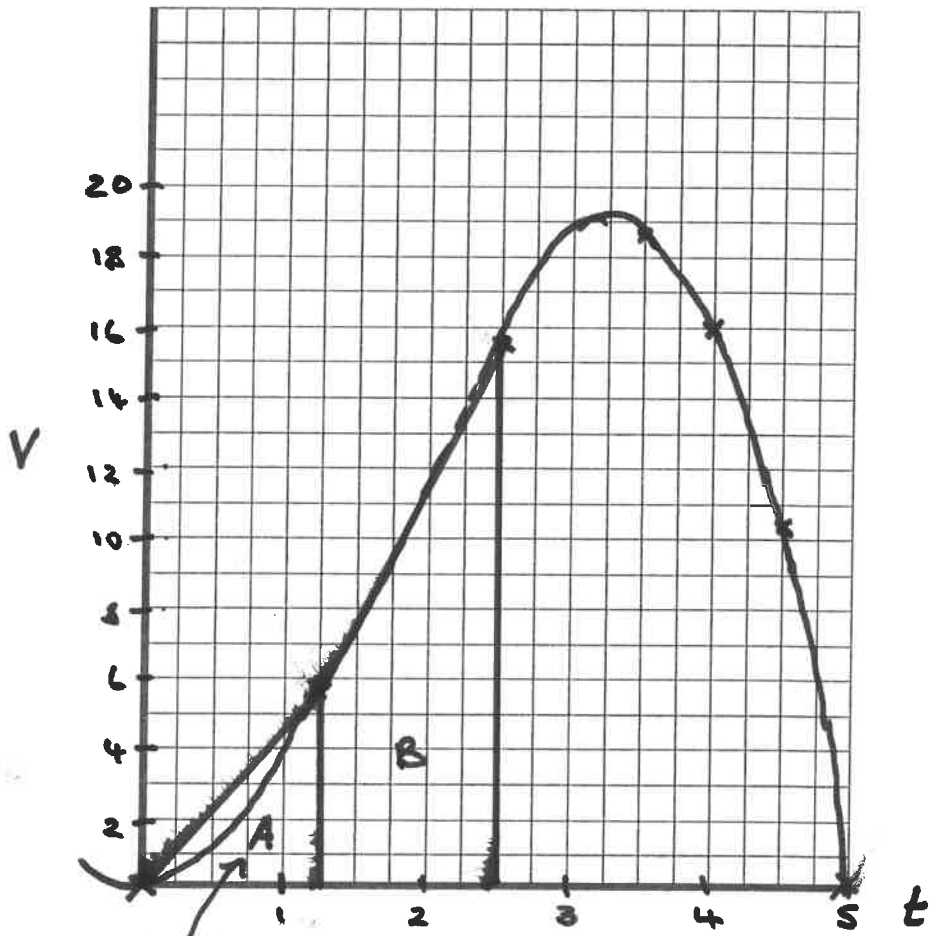
$$v = 5t^2 - t^3$$

[2]

t	0	1.25	2.5	3.5	4	4.5	5
v	0	5·859375	15.625	18·375	16	10·125	0

(b) Using your table plot the graph of the equation for $0 \leq t \leq 5$ seconds.

[3]



(c) Using two equal width trapezia, estimate the area under the graph for $0 \leq t \leq 2.5$

$A = \frac{1·25 \times 5·859375}{2} = \frac{1875}{512}$ $B = \frac{15·625+5·859375}{2} \times 1·25 = 13·41523$

[2]

(d) Will your answer to part (c) be an overestimate or an underestimate?
Make you reason clear.

Over estimate as the curve is under the triangle [1]

(e) Given the area is under a velocity-time graph what does your answer represent? [1]

distance.

(Total 9 marks)

13. The following two cylinders A and B are mathematically similar.

A B

The volume of B is 64 times larger than the volume of A.

(a) If cylinder A has a vertical height of 5cm what is the vertical height of object B?

Scale factor 4.

[2]

$5 \times 4 = 20cm$

(b) The curved surface area of cylinder B is 960cm^2.
What is the radius of cylinder A?

curved surface area $= C \times h.$

[3]

$= \pi d \times h.$

(Total 5 marks)

$960 = \pi d \times 5$ $\quad \pi d = 192$

$d = 61.11549815$

$r = 30.55774907$cm of B.

radius of A: $\dfrac{30.55774907}{4}$

$= 7.639437268$cm

$= 7.6$cm to 1dp.

---

14. A function f(x) is such that ff(x) = x

Given the above fact, tick which of the functions would be correct to represent f(x)?

Possible functions	Only Tick if correct
$f(x) = x + 2$	
$f(x) = x^2$	
$f(x) = {}^1/_x$	✓
$f(x) = \sin(x)$	

$(x+2)+2 = x+4$

$(x^2)^2 = x^4$

$\left(\frac{1}{x}\right) = x$

$\sin(\sin x) \neq x.$

(Total 2 marks)

15. The population of rabbits, P_m, in Yorkshire, at the start of each month, m, is defined as

$P_1 = 10\,000$

$P_{m+1} = 1.05\ P_m$

(a) Complete this list of the Population at the start of the first three months, written as a geometric sequence.

10 000, 10500, 11025 [2]

(b) What will be the population at the start of the tenth month?

16288.94627 [1]

(c) In reality the populations for the first three months are 10 000, 10 600 and 11 236.

With your reasons, what would you suggest should be changed in the original formula to reflect this real data?

$P_{m+1} = 1.06.$ [2]

(6% increase not 5%)

(Total 5 marks)

---

16. Rationalise the denominator for this surd expression,

$$\left(\frac{4+\sqrt{3}}{5-2\sqrt{3}}\right) \times \frac{(5+2\sqrt{3})}{5+2\sqrt{3}} = \frac{20+5\sqrt{3}+8\sqrt{3}+6}{25-12=13}$$

$$= \frac{26+13\sqrt{3}}{13} = 2+\sqrt{3}$$

(Total 2 marks)

---

17 (a) Write the equation $2x^2 + 16x + 40$ in the form $a(x + b)^2 + c$.

$2(x^2+8x+20)$ [3]

$2(x+4)^2+8$

(b) Hence give the coordinates of the minimum point of $y = 2x^2 + 16x + 40$

[2]

$(-4, 8)$

(Total 5 marks)

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $a = 2$, $b = -29$, $c = -63$ $\frac{29 \pm \sqrt{29^2 - 4 \times 2 \times -63}}{4}$

18. Solve this equation giving any answers to 3 significant figures.

$$\frac{4}{x+5} + \frac{5}{x-6} = \frac{2}{3}$$

$= \frac{29 \pm \sqrt{1345}}{4}$

$3 \times \quad 4(x-6) + 5(x+5) = 2(x+5)(x-6)$ $x = 16.4$ or $x = -1.92$

$3(4x - 24 + 5x + 25) = 2(x^2 - x - 30)$

$27x + 3 = 2x^2 - 2x - 60$

$= 2x^2 - 29x - 63.$

(Total 3 marks)

---

19. A bucket holds 1.3 litres to the nearest tenth of a litre, and a cuboid tank measures 60cm x 40cm x 80cm with each length given to the nearest 10cm.

Each time you fill a bucket takes 12 seconds to the nearest second.

What is the minimum amount of time you need to plan for to be **certain** that you can fill the tank?

1.25l. $65 \times 45 \times 85 = 248625\text{cm}^3$

$= 248.625$ l.

$\frac{248.625}{1.25} =$ ~~buckets~~ 198.9

~~seconds~~

98.9×12.5

$= 2486.25$ seconds ~~minutes~~

(Total 4 marks)

---

20. The depth d in metres of water in a river is calculated using the following equation in terms of time in hours.

d = 3 + 2 sin (30t)

(a) What depth is the river after 5 hours?

4

[1]

(b) Helen says "the river is never more than 5m deep". Explain why she is correct.

$5 = 3 + 2\sin(30t)$

[1]

$2 = 2\sin(30t)$ $1 = \sin(30t)$ max!

(c) If the river is 3m deep at 8.00am, when will it next be 3m deep?

Sin is 0 at 180° Check $3 + 2\sin(180) = 3$ [2]

$30t = 180$ $t = 6$. so 8.00am + 6hours = 2pm.

(Total 4 marks)

21. Consider the equation $2x^3 + 3x - 4 = 0$

(a) Show that there is solution to this equation between $x = 0$ and $x = 1$.

$x = 0 \quad y = -4$

$x = 1 \quad y = 2 + 3 - 4 = 1$ } change in sign ⇒ solution. [2]

(b) Show this equation can be rearranged to give $x = \frac{4}{2x^2+3}$

$2x^3 + 3x = 4 \quad x = \frac{4}{2x^2+3}$

$x(2x^2+3) = 4$ [1]

(c) This rearrangement can be written as an iterative formula as follows:

$$x_{n+1} = \frac{4}{2x_n^2 + 3}$$

Using $x_0 = 1$, Find the values of x_1, x_2 and x_3

$x_1 = \frac{4}{2\times1+3} = \frac{4}{5} \qquad x_3 = \frac{4}{2\times\left(\frac{100}{107}\right)^2+3}$ [3]

$x_2 = \frac{4}{2\times\frac{4}{5}^2+3} = \frac{100}{107} \qquad = 0.8426592084$

(Total 6 marks)

TOTAL FOR PAPER IS 80 MARKS